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Strange quark mass from the invariant mass distribution of Cabibbo-suppressed tau decays

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Abstract. Quark mass corrections to the τ hadronic width play a significant role only for the strange quark. The complete determination of τ decays into strange hadronic final states performed by ALEPH allows the extraction of the strange spectral function. New results on strange decay modes from other experiments are also incorporated into the present analysis. Using as input moments of the spectral function the analysis leading to the determination of m_s is conducted using reasonable theoretical constraints on the nonperturbative components. Careful attention is paid to the treatment of the perturbative expansions of the moments which exhibit convergence problems. The result obtained, $m_s(M_\tau^2) = (120 \pm 11_{\rm exp} \pm 8_{V_{us}} \pm 19_{\rm th})$ MeV $= (120\,^{+21}_{-26})$ MeV, is stable over the scale from M_τ^2 down to about 2 GeV^2. Its evolution yields $m_s(1\ {\rm GeV}^2) = (160\,^{+28}_{-35})$ MeV and $m_s(4\ {\rm GeV}^2) = (116\,^{+20}_{-25})$ MeV.

1 Introduction

The high precision data on τ decays [1,2] collected at LEP and CESR provide a powerful tool to investigate the dynamics of strong interaction at low energies and to determine basic parameters of the theory. The QCD analysis of the nonstrange inclusive τ decay width [3–8] has led to accurate measurements [9–12] of the strong coupling constant at the τ mass scale, $\alpha_s(M_\tau^2)$, which complements and competes in accuracy with the high precision determination of $\alpha_s(M_Z^2)$ from measurements of the Z width at LEP.

More recently, experimental studies of the Cabibbo-suppressed width of the τ became available [13,14], allowing to initiate a systematic study of the corrections induced by the strange quark mass in the τ decay width [3, 15–22]. From the separate measurement of the strangeness $S = 0$ and $S = -1$ τ decay widths[1] it is possible to pin down the SU(3) breaking effects and to perform a reliable determination of the strange quark mass.

The ALEPH data [13] have been used in previous analyses to extract the value of $m_s(M_\tau^2)$, leading in some cases to different, albeit not inconsistent results [13,19,20,22]. In this paper we present an updated common analysis [23], taking into account recent experimental information [24–26] in addition to the ALEPH data. Particular attention is given to the analysis of theoretical uncertainties and to the stability of the results.

Even within the relatively large statistical errors of the present data, the strange quark mass determination from τ decays has already achieved an accuracy good enough to substantially reduce the range quoted by the Particle Data Group [24].

2 Theoretical framework

The theoretical analysis of the hadronic τ decay width involves the two-point correlation functions

$$\Pi^{\mu\nu}_{ij,V}(q) \equiv i\int {\rm d}^4x\, e^{iqx}\, \langle 0|T\left\{V^\mu_{ij}(x)\, V^\nu_{ij}(0)^\dagger\right\}|0\rangle\,, \quad (1)$$

$$\Pi^{\mu\nu}_{ij,A}(q) \equiv i\int {\rm d}^4x\, e^{iqx}\, \langle 0|T\left\{A^\mu_{ij}(x)\, A^\nu_{ij}(0)^\dagger\right\}|0\rangle\,, \quad (2)$$

associated with the time-ordered vector, $V^\mu_{ij}(x) \equiv \overline{q}_j\gamma^\mu q_i$, and axial-vector, $A^\mu_{ij}(x) \equiv \overline{q}_j\gamma^\mu\gamma_5 q_i$, colour-singlet quark currents. The subscripts i, j denote the corresponding light quark flavours (up, down, and strange). The correlators admit the Lorentz decompositions

$$\Pi^{\mu\nu}_{ij,V/A}(q) = \left(-g^{\mu\nu}q^2 + q^\mu q^\nu\right)\Pi^T_{ij,V/A}(q^2) + q^\mu q^\nu\,\Pi^L_{ij,V/A}(q^2)\,, \quad (3)$$

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[1] Throughout this paper, charge conjugate states are implied.

where the superscript in the transverse and longitudinal components denotes the corresponding spin, $J = 1$ (T) and $J = 0$ (L), in the hadronic rest frame.

The hadronic decay rate of the τ lepton,

$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \text{hadrons}\,\nu_\tau(\gamma)\right)}{\Gamma\left(\tau^- \to e^-\,\overline{\nu}_e\,\nu_\tau(\gamma)\right)} = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}\,, \qquad (4)$$

can be expressed as an integral of the spectral functions $\mathrm{Im}\,\Pi^T(s)$ and $\mathrm{Im}\,\Pi^L(s)$ over the invariant mass s of the final-state hadrons, where

$$\Pi^J(s) \equiv |V_{ud}|^2\left[\Pi^J_{ud,V}(s) + \Pi^J_{ud,A}(s)\right] + |V_{us}|^2\left[\Pi^J_{us,V}(s) + \Pi^J_{us,A}(s)\right]\,, \qquad (5)$$

with $|V_{ij}|$ the corresponding Cabibbo-Kobayashi-Maskawa (CKM) quark mixing factors. In (4), $R_{\tau,V}$ and $R_{\tau,A}$ correspond to the two terms proportional to $|V_{ud}|^2$ in (5) and $R_{\tau,S}$ contains the remaining Cabibbo-suppressed contributions.

Using the analytic properties of $\Pi^J(s)$, one can express R_τ as a contour integral in the complex s-plane running counter-clockwise around the circle $|s| = M_\tau^2$,

$$R_\tau = -\pi i \oint_{|s|=M_\tau^2} \frac{\mathrm{d}s}{s}\left(1 - \frac{s}{M_\tau^2}\right)^3 \times \left\{3\left(1 + \frac{s}{M_\tau^2}\right) D^{L+T}(s) + 4\,D^L(s)\right\}\,. \qquad (6)$$

We have used integration by parts to rewrite R_τ in terms of the logarithmic derivatives of the relevant correlators,

$$D^{L+T}(s) \equiv -s\frac{\mathrm{d}}{\mathrm{d}s}\left[\Pi^{L+T}(s)\right]\,, \qquad (7)$$

$$D^L(s) \equiv \frac{s}{M_\tau^2}\,\frac{\mathrm{d}}{\mathrm{d}s}\left[s\,\Pi^L(s)\right]\,, \qquad (8)$$

which satisfy homogeneous renormalization group equations (RGE). This eliminates unwanted renormalization-scheme dependent subtraction constants which do not contribute to any physical observable.

For large enough $-s$, the contributions to $D^J(s)$ can be organized using the Operator Product Expansion (OPE) in a series of local gauge-invariant scalar operators of increasing dimension $D = 2n$, times the appropriate inverse powers of $-s$. This expansion is expected to be well behaved along the complex contour $|s| = M_\tau^2$, except for the crossing point with the positive real axis [27]. As shown in (6), the region near the physical cut is strongly suppressed by a zero of order three at $s = M_\tau^2$. Therefore, the uncertainties associated with the use of the OPE near the time-like axis are expected to be small. Inserting the OPE in (6) and evaluating the contour integral, one can rewrite R_τ as an expansion in inverse powers of M_τ^2 [3]. This leads to a rigorous prediction for R_τ and its different flavour components, which, in particular, allows $\alpha_s(M_\tau^2)$ to be accurately determined [9–12].

The measurement of the invariant mass distribution of the final state hadrons provides additional information on the QCD dynamics. The moments [7]

$$R_\tau^{kl} \equiv \int_0^{M_\tau^2} ds\,\left(1 - \frac{s}{M_\tau^2}\right)^k\left(\frac{s}{M_\tau^2}\right)^l\frac{dR_\tau}{ds}\,, \qquad (9)$$

can be calculated theoretically in the same way as $R_\tau \equiv R_\tau^{00}$. The corresponding contour integrals take the form

$$R_\tau^{kl} = -\pi i \oint_{|x|=1}\frac{\mathrm{d}x}{x}\left\{3\,\mathcal{F}^{kl}_{L+T}(x)\,D^{L+T}(M_\tau^2 x) + 4\,\mathcal{F}^{kl}_L(x)\,D^L(M_\tau^2 x)\right\}\,, \qquad (10)$$

where all kinematical factors have been absorbed into the kernels $\mathcal{F}^{kl}_{L+T}(x)$ and $\mathcal{F}^{kl}_L(x)$. Their explicit expressions were given in [19]. Performing the contour integration, the result can be written as

$$R_\tau^{kl} \equiv 3\left[|V_{ud}|^2 + |V_{us}|^2\right] S_{\mathrm{EW}}\left\{1 + \delta'_{\mathrm{EW}} + \delta^{(0)} + \sum_{D=2,4,\cdots}\left(\cos^2\theta_C\,\delta^{kl(D)}_{ud} + \sin^2\theta_C\,\delta^{kl(D)}_{us}\right)\right\}, \qquad (11)$$

where $\sin^2\theta_C \equiv |V_{us}|^2/[|V_{ud}|^2 + |V_{us}|^2]$ and where we have pulled out the electroweak radiative corrections $S_{\mathrm{EW}} = 1.0194 \pm 0.0040$ [28] and $\delta'_{\mathrm{EW}} \simeq 0.0010$ [29].

The dimension-zero contribution $\delta^{(0)}$ is the purely perturbative correction neglecting quark masses, which, owing to chiral symmetry, is identical for the vector and axial-vector parts. The symbols $\delta^{kl(D)}_{ij} \equiv [\delta^{kl(D)}_{ij,V} + \delta^{kl(D)}_{ij,A}]/2$ stand for the average of the vector and axial-vector contributions from dimension $D \geq 2$ operators; they contain an implicit suppression factor $1/M_\tau^D$.

A general analysis of the relevant $\delta^{00(D)}_{ij,V/A}$ contributions was presented in [3]. A detailed study of the perturbative piece $\delta^{(0)}$ was later performed in [6], where a resummation of higher-order corrections induced by running effects along the integration contour —denoted Contour-Improved Fixed-Order Perturbation Theory ($\mathrm{FOPT_{CI}}$)— was achieved with renormalization-group techniques.

The leading quark-mass corrections of dimension two have been studied in [3,15–19]; these contributions are the dominant SU(3) breaking effect, which generates the wanted sensitivity to the strange quark mass. The separate measurement of the Cabibbo-allowed and Cabibbo-suppressed decay widths of the τ allows one to pin down this SU(3) breaking effect through the differences [13]

$$\delta R_\tau^{kl} \equiv \frac{R^{kl}_{\tau,V+A}}{|V_{ud}|^2} - \frac{R^{kl}_{\tau,S}}{|V_{us}|^2} = 3\,S_{EW}\sum_{D\geq 2}\left[\delta^{kl\,(D)}_{ud} - \delta^{kl\,(D)}_{us}\right]\,. \qquad (12)$$

These observables vanish in the SU(3) limit, which helps to reduce many theoretical uncertainties. In particular they are free of possible (flavour-independent) instanton and/or

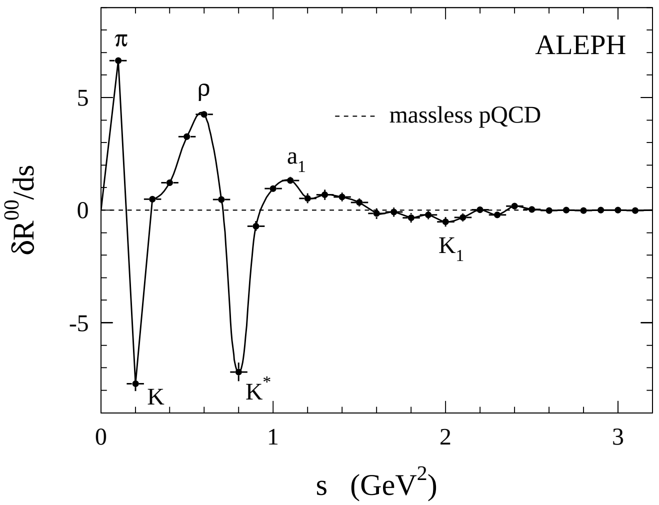

Fig. 1. Integrand of (9) with (k=0, l=0) for the difference (12) of the Cabibbo-corrected nonstrange and strange invariant mass spectra. The contribution from massless perturbative QCD (pQCD) vanishes. To guide the eye, the solid line interpolates between the bins of constant 0.1 GeV^2 width

renormalon contributions which could mimic dimension-two corrections. A detailed theoretical analysis of the leading $D = 2$ and $D = 4$ contributions to (12) has been given in [19].

3 Experimental data

ALEPH has published a comprehensive study of τ decay modes including kaons (charged, K^0_S and K^0_L) [13,30–32] with up to four hadrons in the final state. Such an analysis is necessary in order to separate $S = -1$ and $S = 0$ modes with a $K\overline{K}$ pair.

The total branching ratio for τ decay into strange final states is measured to be $B_S = (2.87 \pm 0.12)\%$ corresponding to

$$R_{\tau,S} = 0.1610 \pm 0.0066\,, \qquad (13)$$

using the combined value for the electronic branching ratio, $B_e = (17.794 \pm 0.045)\%$, obtained from the measurements of the leptonic branching ratios and of the τ lifetime [24], assuming lepton universality. Since the QCD expectation for a massless quark is 0.1809 ± 0.0036, the result (13) is evidence for a massive strange quark.

The strange spectral function is derived from the distribution of the invariant hadronic mass. It is dominated at low mass by the K pole and the $K^*(892)$ resonance and at larger masses by the axial-vector resonances $K_1(1270)$ and $K_1(1400)$, decaying into $\overline{K}\pi\pi$ final states.

Figure 1 shows the weighted integrand of the lowest moment δR^{00}_τ from the ALEPH data, as a function of the invariant mass-squared, and for which the expectation from perturbative QCD (pQCD) vanishes in the case of massless quarks.

The present analysis takes into account recent branching ratio results obtained by CLEO [25] and OPAL [26],

Table 1. Measured branching ratios (10^{-3}) for τ decays into strange final states $+\nu_\tau$. The ALEPH values are from [13] and the world averages (including ALEPH) from [24–26]. The branching ratios for the $(\overline{K}4\pi)^-$ and $(\overline{K}5\pi)^-$ modes are estimated from the measured branching ratios for the 5π and 6π modes introducing Cabibbo and phase space suppression.

Mode	ALEPH	World Average
K^-	6.96 ± 0.29	6.81 ± 0.23
$K^-\pi^0$	4.44 ± 0.35	4.49 ± 0.34
$\overline{K}^0\pi^-$	9.17 ± 0.52	8.78 ± 0.38
$K^-\pi^0\pi^0$	0.56 ± 0.25	0.58 ± 0.24
$\overline{K}^0\pi^-\pi^0$	3.27 ± 0.51	3.62 ± 0.40
$K^-\pi^+\pi^-$	2.14 ± 0.47	2.76 ± 0.48
$K^-\eta$	0.29 ± 0.14	0.27 ± 0.06
$(\overline{K}3\pi)^-$	0.76 ± 0.44	0.62 ± 0.34
$K_1(1270)^- \to K^-\omega$	0.67 ± 0.21	0.67 ± 0.21
$(\overline{K}4\pi)^-$ (estim.)	0.34 ± 0.34	0.34 ± 0.34
$(\overline{K}5\pi)^-$ (estim.)	0.06 ± 0.06	0.06 ± 0.06
Sum	28.65 ± 1.18	29.00 ± 1.02

Table 2. Measured spectral moments δR^{kl}_τ (top table: first error is experimental, second from $|V_{us}|$) and their experimental correlations (bottom table).

(k,l)	δR^{kl}_τ
(0,0)	$0.374 \pm 0.118 \pm 0.062$
(1,0)	$0.398 \pm 0.065 \pm 0.042$
(2,0)	$0.399 \pm 0.044 \pm 0.031$
(3,0)	$0.396 \pm 0.034 \pm 0.024$
(4,0)	$0.395 \pm 0.028 \pm 0.020$

(k,l)	(0,0)	(1,0)	(2,0)	(3,0)	(4,0)
(0,0)	1	0.94	0.83	0.71	0.61
(1,0)	-	1	0.97	0.90	0.82
(2,0)	-	-	1	0.98	0.94
(3,0)	-	-	-	1	0.99
(4,0)	-	-	-	-	1

thus improving the normalization of the individual contributions to the ALEPH strange spectral function.

The largest effect is found in the $\overline{K}\pi\pi$ final states. In particular, the branching ratio for the $K^-\pi^+\pi^-$ mode, $(2.14 \pm 0.47) \times 10^{-3}$ for ALEPH, becomes in the average $(2.76 \pm 0.48) \times 10^{-3}$, where the inflated error takes into account the poor χ^2 of the fit. Also, the branching ratio for $\overline{K}^0\pi^-\pi^0$, $(3.27 \pm 0.51) \times 10^{-3}$ for ALEPH, becomes $(3.62 \pm 0.40) \times 10^{-3}$ in the average.

The resulting world average for the total strange rate is found to be

$$R_{\tau,S} = 0.1630 \pm 0.0057\,, \qquad (14)$$

with a 13% improvement in precision. Table 1 summarizes the different contributions to $R_{\tau,S}$. The value for $|V_{us}|$ is taken from the Particle Data Group unitarity fit of the CKM matrix [24] yielding $|V_{us}| = 0.2225 \pm 0.0021$. The experimental results for the spectral moments δR_τ^{kl} and their correlations are given in Table 2. It can be observed that the central values stay rather constant between $k = 0$ and $k = 4$, while the errors decrease. This is due to the fact that for higher k values the spectral moments are relying more and more on the accurately measured K and $K^*(892)$ channels. The contributions from $K\,n\pi$, $n \geq 2$ modes to the moments are negligible for $k > 2$. The error from $|V_{us}|$ is also reduced as $R_{\tau,S}^{k0}$ decreases with k. The contributions from the various decay modes are visualized in Fig. 2.

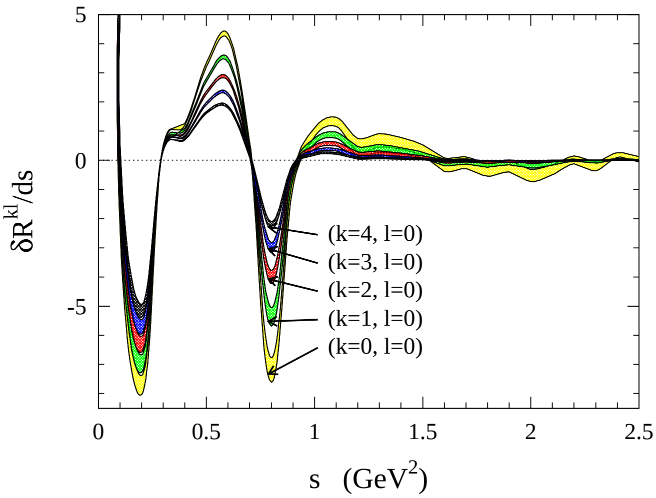

Fig. 2. Integrand of (9) for the moments $k = 0, \ldots, 4$ and $l = 0$ of the difference (12). The widths of the interpolating bands corresponds to one standard deviation

4 Phenomenological analysis

The present analysis is performed using the general OPE framework including the light quark masses, the $D = 4$ quark mass corrections and $D = 6$ nonperturbative contributions [3], neglecting higher dimensions. However, these terms are numerically insignificant compared to the present experimental uncertainty so that for all practical purposes the strange quark mass is obtained from the relation:

$$m_s^2(M_\tau^2) \simeq \frac{M_\tau^2}{\Delta_{kl}^{(2)}(a_\tau)} \left[\frac{\delta R_\tau^{kl}}{24 S_{EW}} + 2\pi^2 \frac{\langle \delta O_4(M_\tau^2) \rangle}{M_\tau^4} Q_{kl}(a_\tau) \right], \quad (15)$$

where $\Delta_{kl}^{(2)}(a_\tau)$ and $Q_{kl}(a_\tau)$ are the pQCD series, defined in [19], associated with the $D = 2$ and $D = 4$ contributions to δR_τ^{kl}, $a_\tau = \alpha_s(M_\tau^2)/\pi$, and

$$\langle \delta O_4(M_\tau^2) \rangle \equiv \langle 0 | m_s\, \bar{s}s - m_d\, \bar{d}d | 0 \rangle (M_\tau^2) \simeq -(1.5 \pm 0.4) \times 10^{-3}\ \mathrm{GeV}^4 . \quad (16)$$

Unfortunately, the QCD series for $\Delta_{kl}^{(2)}$ turn out to be problematic. They exhibit bad convergence originating from the asymptotic behaviour of their longitudinal components [17–19]. While the longitudinal part is known to third order, the $L+T$ series has been calculated to second order only. We estimate the complete third order contributions (dominated by the badly converging, but known longitudinal part) by assuming a geometrical growth of the perturbative coefficients, $c_3^{L+T} \simeq c_2^{L+T}(c_2^{L+T}/c_1^{L+T}) \simeq 323$, of the corresponding Adler function [19]. The value of the strong coupling constant, $\alpha_s(M_\tau^2) = 0.334 \pm 0.022$, is taken from the analyses [11,12] of the nonstrange $V+A$ moments R_τ^{kl}. Correlations between this value of $\alpha_s(M_\tau^2)$ and the moments $\delta R_{kl}^{(2)}$ are negligible since the uncertainty on the former is dominated by theory and the latter by the measurement of the strange component. Using these inputs, the pQCD series $\Delta_{kl}^{(2)}$ can be displayed up to third order using $\mathrm{FOPT_{CI}}$[6], for example:

$$\begin{aligned}
\Delta_{00}^{(2)}(a_\tau) &= 0.9734 + 0.4811 + 0.3718 + 0.3371 + \ldots \\
\Delta_{10}^{(2)}(a_\tau) &= 1.0390 + 0.5576 + 0.4820 + 0.4771 + \ldots \\
\Delta_{20}^{(2)}(a_\tau) &= 1.1154 + 0.6432 + 0.6082 + 0.6470 + \ldots \\
\Delta_{30}^{(2)}(a_\tau) &= 1.1990 + 0.7374 + 0.7516 + 0.8507 + \ldots \\
\Delta_{40}^{(2)}(a_\tau) &= 1.2880 + 0.8404 + 0.9142 + 1.0928 + \ldots
\end{aligned} \quad (17)$$

The exhibited behaviour is that of asymptotic series close to their point of "minimum sensitivity" and a prescription is needed to evaluate the expansions and to reasonably estimate their uncertainties. Close examination of examples of mathematical asymptotic series suggests that a reasonable procedure is to truncate the expansion where the terms reach their minimum value. The precise prescription —cutting at the minimum, or one order before, including the full last term or only a fraction of it— is somewhat arbitrary and this ambiguity must be reflected by a specific uncertainty attached to the procedure. In this analysis we adopt as a rule keeping all terms up to (and including) the minimal one and assigning as a systematic uncertainty the full value of the last term retained. It follows that the $\Delta_{k0}^{(2)}$ series are then summed up to third order for $k = 0, 1$, second order for $k = 2$ and first order for $k = 3, 4$. It can be remarked that the assigned truncation uncertainty is numerically equivalent to quoting an uncertainty of 330 on x_3^{L+T}, *i.e.*, a 200% error.

We disregard moments (k, l) with $l \neq 0$ since they suffer from an increased dependence on the $D \geq 6$ nonperturbative terms. In addition such moments carry little information on m_s.

The numerical values of the $D = 4$ perturbative series Q_{kl} are given in Table 3. The first errors give the estimated theoretical uncertainties from missing higher order terms in the (fast converging) expansion and the second errors

Table 3. Numerical values of the relevant $D = 4$ perturbative expansions for $\alpha_s(M_\tau^2) = 0.334 \pm 0.022$. The first errors give the estimated theoretical uncertainties from missing higher order terms in the (fast converging) series and the second errors quote the changes induced by the uncertainty in the strong coupling.

(k,l)	$Q_{kl}(a_\tau)$
(0,0)	$1.07 \pm 0.02 \pm 0.01$
(1,0)	$1.50 \pm 0.02 \pm 0.01$
(2,0)	$1.92 \pm 0.01 \pm 0.00$
(3,0)	$2.33 \pm 0.01 \pm 0.01$
(4,0)	$2.72 \pm 0.03 \pm 0.02$

Table 4. The strange quark mass at M_τ determined from each of the $\delta R_\tau^{k,l}$ experimental moments. The breakdown of the different sources of uncertainties corresponds to: experimental, $|V_{us}|$, $\alpha_s(M_\tau^2)$, quark condensates, truncation of the perturbative series $\Delta_{k0}^{(2)}(a_\tau)$, and renormalization scale. The last column gives the total theoretical uncertainty excluding the contribution from $|V_{us}|$. Errors have been symmetrized for reading convenience.

(k,l)	m_s (MeV)	σ_{m_s} (MeV)						
		exp.	$\lvert V_{us}\rvert$	α_s	$\langle m_s\bar{s}s\rangle$	trunc.	R-scale	th.
(0,0)	132	26	13	2	4	9	9	14
(1,0)	120	13	9	3	4	10	11	16
(2,0)	117	10	7	3	6	14	14	21
(3,0)	117	9	8	2	8	19	16	27
(4,0)	103	7	5	3	9	20	19	29

quote the changes induced by the uncertainty in the strong coupling.

5 Results and discussion

With the prescription given in the preceding section for the perturbative expansion $\Delta_{kl}^{(2)}(a_\tau)$ and the data from ALEPH, we first derive the strange quark mass from each experimental δR_τ^{k0} moment. The results are quoted in Table 4. The values obtained for $m_s(M_\tau^2)$ are rather stable between $k = 0$ and $k = 4$. There is however a small decrease which could be of statistical nature, but could also indicate a deterioration of the validity of the OPE, since larger k values emphasize the low-mass contributions rendering the approach less inclusive. The breakdown of the error on $m_s(M_\tau^2)$ into its contributions is given in Table 4: whereas the experimental uncertainty dominates at small k, the theoretical uncertainty, which receives its main contribution from the truncation of the perturbative series (17) and the renormalization scale, increases with k and dominates for $k \geq 1$. To estimate the latter uncertainty, the renormalization scale is varied from $0.75M_\tau$ to $2M_\tau$. All the theoretical errors are added in quadrature. Since the square of the strange quark mass is measured (*c.f.*, 15), the errors on $m_s(M_\tau^2)$ are asymmetric. For reading convenience, they are symmetrized throughout this paper, except for the final result.

The fact that error contributions are given for the truncation and the renormalization scale, both related to the limited number of terms in the perturbative expansion, can be considered to be a conservative approach. In addition, one can check the quoted systematic uncertainty by modifying the truncation procedure: cutting off the expansion one order less than the minimum term yields $m_s(M_\tau^2)$ values ranging from 143 MeV (for $k = 0$) to 127 MeV (for $k = 4$), showing a slightly better relative stability, but deviating from the nominal results given in Table 4 by values consistent with the quoted truncation errors. Another test of the handling of the perturbative series and its poor convergence is obtained by comparing the chosen contour-improved method ($\mathrm{FOPT_{CI}}$) to the more standard procedure of fixed order expansion without partial resummation (FOPT). The latter method provides $m_s(M_\tau^2)$ values in the range 121 MeV ($k = 0$) to 119 MeV ($k = 4$), *i.e.*, remarkably stable and consistent with the nominal values within the uncertainties relevant to the treatment of the perturbative series.

The $|V_{us}|$ uncertainty is never dominant for any value of k. Other sources of systematic effects are negligible: in particular, the uncertainties from S_{EW} and higher order nonperturbative operators $\langle \delta O_6 \rangle$ (using the estimate of [19]) lie in the range 0.3 to 0.6 MeV.

To optimize experimental and theoretical sensitivities, a combined fit is performed using several moments. The overall sensitivity increases up to $k = 2$ while no significant improvement occurs for larger k values and the fitted $m_s(M_\tau^2)$ value remains stable. Keeping only lower k moments is also justified in view of a possible breakdown of the OPE for higher moments and their worse perturbative convergence (*c.f.*, (17)). The constrained fit of the $k = 0, 1, 2$ moments takes into account the very large correlations between the experimental values and yields the result:

$$\begin{aligned} m_s(M_\tau^2) &= (120 \pm 11_{\mathrm{exp}} \pm 8_{V_{us}} \pm 19_{\mathrm{th}})\ \mathrm{MeV} \\ &= (120\,^{+21}_{-26})\ \mathrm{MeV}\,, \end{aligned} \tag{18}$$

where the theoretical error includes an additional uncertainty of $\sigma_{\mathrm{OPE}} = 12$ MeV which accounts for the stability of the result when using different moments. For reading convenience, the errors in the first line of (18) have been symmetrized.

The result (18) can be evolved to different scales using the four-loop RGE γ-function [33], yielding

$$m_s(1\ \mathrm{GeV}^2) = (160\,^{+28}_{-35})\ \mathrm{MeV}\,, \tag{19}$$

$$m_s(4\ \mathrm{GeV}^2) = (116\,^{+20}_{-25})\ \mathrm{MeV}\,. \tag{20}$$

As shown in the ALEPH analysis of the nonstrange τ decays [11], one can test the validity of the QCD analysis by simulating the physics of a hypothetical τ lepton of lower mass, $\sqrt{s_0}$. This is obtained by replacing M_τ^2 by s_0 everywhere in (6) and (9), and correcting the latter for the modified kinematic factor. Under the assumption

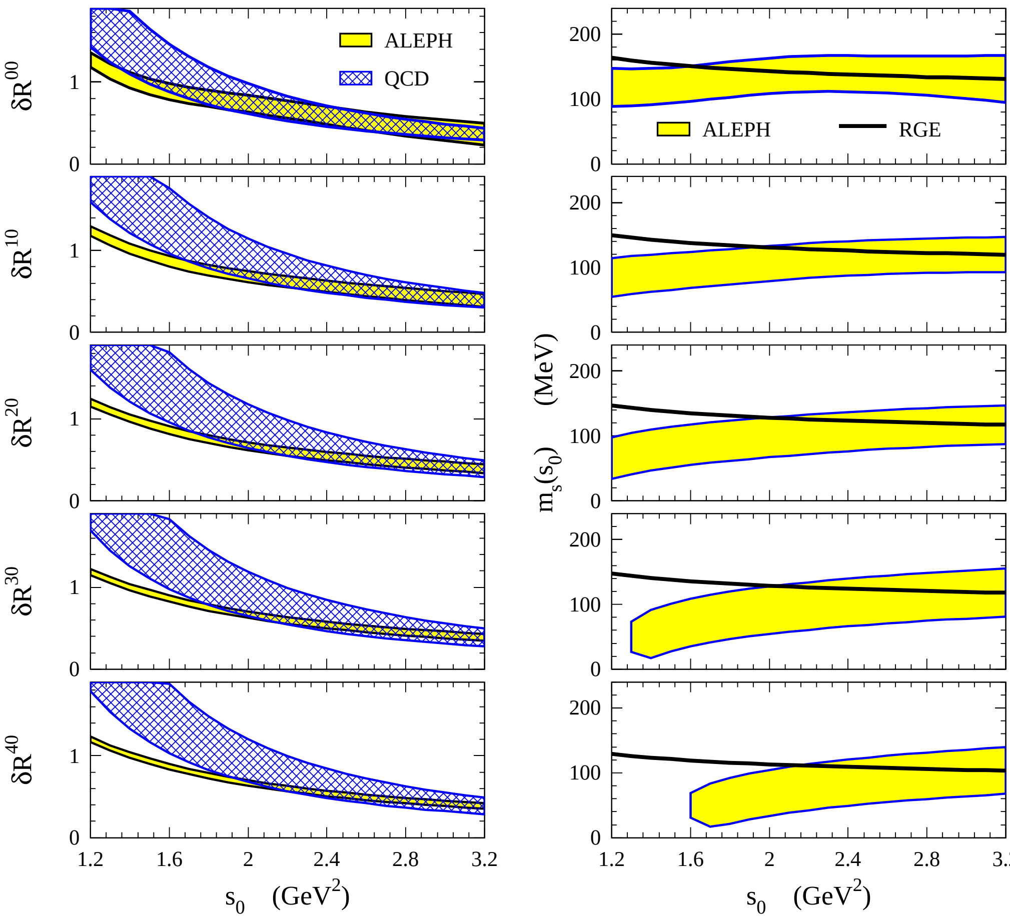

Fig. 3. The observables $\delta R_\tau^{k0}(s_0)$ as a function of the "τ-mass" squared s_0 confronted to the QCD predictions and the derived $m_s(s_0)$ compared to the QCD RGE running. The two bands correspond to the experimental and theoretical uncertainties. By construction, data and theory agree at $s_0 = M_\tau^2$

of quark-hadron duality, the evaluation of the observables as function of s_0 constitutes a test of the OPE approach, since the energy dependence of the theoretical predictions is determined once the parameters of the theory are fixed. The results of this exercise are given in Fig. 3, showing the variation with s_0 of the first four δR_τ^{k0} moments and the value for $m_s(s_0)$ derived from each moment. The bands indicate the experimental and theoretical uncertainties. The agreement between data and theory, perfect at $s_0 = M_\tau^2$ by construction, remains acceptable for lower s_0 values, down to 1.6 (2.4) GeV^2 for $k = 0$ (4). For the first three moments used in the final determination, the running observed in data follows the RGE evolution down to about 2 GeV^2. It should be pointed out that the s_0 dependence of the theoretical prediction is obtained following the truncation method defined in Sect. 4 and applied at $s_0 = M_\tau^2$. If the same rule had been consistently used at each s_0 point a better agreement would have been found down to much lower s_0 values, at the price of introducing steps in the prediction, corresponding to dropped terms in the expansion (according to the prescription given in Sec. 4). This observation provides another consistency test of the procedure.

6 Comparison with other analyses

Several analyses of the ALEPH strange spectral function have been performed in order to extract $m_s(M_\tau^2)$. In the ALEPH paper [13], a very conservative road was followed, using only the $L+T$ part since its perturbative expansion converges well. A price was paid in the experimental sensitivity, yielding rather large uncertainties on the result. Also, the experimental moments were fitted, not only to the strange quark mass, but also to the values of the nonperturbative operators $\langle \delta O_{6,8} \rangle$. The values obtained were in reasonable agreement with our assumption in the present work, but unfortunately created an unlucky shift to a larger m_s value. As we have discussed here it is safe to neglect such contributions resulting in a more constraining fit. The value found by ALEPH is $m_s(M_\tau^2) = (176\,{}^{+37_{\rm exp}}_{-48_{\rm exp}}\,{}^{+24_{\rm th}}_{-28_{\rm th}} \pm 14_{\rm meth})$ MeV, on the high side of the present determination for the reasons analyzed above. The third quoted error covers uncertainties in the fit and in the experimental separation of $J = 0,1$ states necessary to work with only the $L+T$ part.

The method used in [19] is rather close to the present one, with the result $m_s(M_\tau^2) = (119 \pm 12_{\rm exp} \pm 10_{V_{us}} \pm 18_{\rm th})$

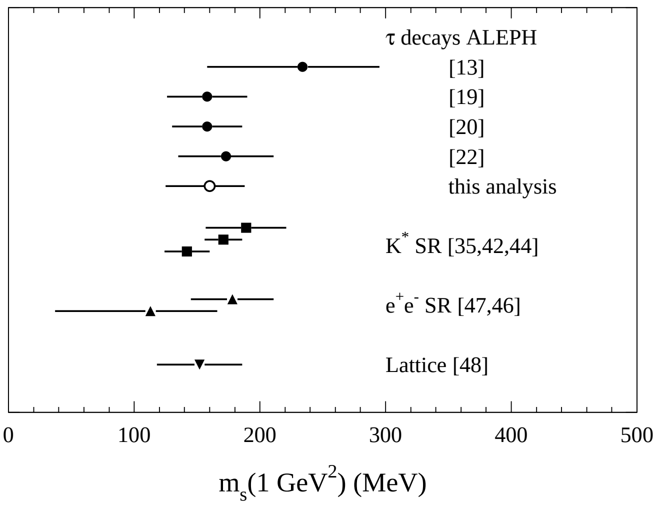

Fig. 4. The determination of $m_s(1\ \mathrm{GeV}^2)$ in this work compared to the results of other analyses based on the ALEPH strange spectral function and on other approaches. Details are given in the text (SR = sum rules)

MeV. Apart from using better experimental moments, the improvements for the new analysis presented here deals with a better treatment of the perturbative series for the different moments and an optimal use of data and theory through a constrained fit of three moments.

The analysis of [22] uses the ALEPH δR_τ^{00} moment and obtains $m_s(M_\tau^2) = (130 \pm 27_{\mathrm{exp}} \pm 9_{\mathrm{th}})$ MeV. It advocates contour-improved resummation and employs an effective charge as well as effective masses absorbing the higher perturbative terms. The central value agrees with the corresponding result of the present analysis (given in the first line of Table 4). The quoted theoretical uncertainty is half as small as the one derived here, but not justified in the paper. Apparently no uncertainty is included from $|V_{us}|$, which should be ± 13 MeV.

Finally, the last analysis [20] using the ALEPH data makes use of weight functions multiplying the correlators in (6). These weights are tuned to improve the convergence of the perturbative series, while suppressing the less accurate high-mass part of the strange spectral function. This latter feature is close to our use of higher moments in k. Since we have observed some deterioration of the convergence for these moments and correspondingly a systematic shift of the derived m_s values, it is not completely clear to what extent this procedure is applicable and thus how reliable the answer is. Nevertheless their result is in good agreement with (18), with a smaller theoretical uncertainty, apparently not including the effect from the arbitrary choice of the renormalization scale. When evolved to M_τ^2, the result of [20] reads $m_s(M_\tau^2) = (119 \pm 14_{\mathrm{exp}} \pm 12_{V_{us}} \pm 10_{\mathrm{th}})$ MeV.

Other determinations of m_s have been obtained by analyses of the divergence of the vector and axial-vector current two-point function correlators [34–44]. The phenomenological information on the associated scalar and pseudoscalar spectral functions is reconstructed from phase-shift resonance analyses which are yet incomplete over the considered mass range and need to be supplemented by other assumed ingredients, in particular the description of the continuum.

Another approach [45] considers the difference between isovector and hypercharge vector current correlators as related to the $I = 1$ and $I = 0$ spectral functions accessible in e^+e^- annihilation into hadrons at low energy. The author obtains $m_s(1\ \mathrm{GeV}^2) = (198 \pm 29)$ MeV. This approach was criticized in [46], pointing out the possibility of large isospin breaking leading to a significant deviation for the extracted m_s value. Afterwards, in [47], new SU(3)-breaking sum rules much less affected by SU(2) breaking were studied, with the result $m_s(1\ \mathrm{GeV}^2) = (178 \pm 33)$ MeV.

Finally, lattice QCD calculations of m_s are available (see, *e.g.*, [48,49] for recent reviews and references therein), whose results are quite spread at present. The average result reads [48] $m_s(4\ \mathrm{GeV}^2) = (110 \pm 25)$ MeV.

The m_s determinations discussed above are compared in Fig. 4 at the scale of 1 GeV^2.

The sum of the up and down quark masses has been determined with Finite Energy Sum Rules [50] with the result $(m_u + m_d)(1\ \mathrm{GeV}^2) = (12.8 \pm 2.5)$ MeV. Using the ratio $2m_s/(m_u + m_d) = 24.4 \pm 1.5$, obtained within $O(p^4)$ Chiral Perturbation Theory and the large N_C limit [51], this result is in nice agreement with the present determination (19).

7 Conclusions

We have used the strange spectral function in τ decays measured by ALEPH to extract the strange quark mass. The normalization has been improved by incorporating recent branching ratio determinations from CLEO and LEP experiments. The stability of the m_s result has been checked by using several moments of the invariant mass distribution. The final value is obtained from a constrained fit of 3 moments, yielding

$$\begin{aligned} m_s(M_\tau^2) &= (120 \pm 11_{\mathrm{exp}} \pm 8_{V_{us}} \pm 19_{\mathrm{th}})\ \mathrm{MeV} \\ &= (120^{+21}_{-26})\ \mathrm{MeV}. \end{aligned}$$

The theoretical error accounts for uncertainties, *(i)*, in the perturbative series used to $O(\alpha_s^3)$ or lower, following a prescription for truncating asymptotic series, *(ii)*, in the OPE approach, and *(iii)*, from smaller sources.

At the customary scales where quark masses are quoted, this result becomes

$$\begin{aligned} m_s(1\ \mathrm{GeV}^2) &= (160\,^{+28}_{-35})\ \mathrm{MeV}\,, \\ m_s(4\ \mathrm{GeV}^2) &= (116\,^{+20}_{-25})\ \mathrm{MeV}\,. \end{aligned}$$

The stability of the result is tested by varying the mass of a hypothetical τ lepton using the measured spectral function. The analysis remains reliable within the given errors down to a mass of about 1.4 GeV. The procedure used for truncating the asymptotic QCD series would in fact improve the stability down to even smaller masses, with however less accuracy.

The uncertainty on the m_s result is dominated by theory. This does not mean that more precise data on Cabibbo-suppressed τ decays are not necessary. In particular the quoted error on the validity of the OPE is derived from the variation of m_s extracted from moments of increasing orders. It is not clear if the observed effect is of statistical nature and would not disappear with larger data samples. In general, more data, as expected from the B-factories presently in operation, will allow more checks to be performed with a possible gain in the theoretical uncertainty [52]. Thus future measurements could permit a more precise determination of the strange quark mass along the lines presented here.

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